

Student Workbook

Modeling Using Variation

Name: _____

Date: _____

Period: _____

Learning Target: I can write and use direct, inverse, and joint variation models to solve real-world problems.

Vocabulary

Term	Meaning
Direct Variation	$y = kx$ — as x increases, y increases proportionally.
Inverse Variation	$y = k/x$ — as x increases, y decreases proportionally.
Joint Variation	$z = kxy$ — z varies directly as the product of two (or more) variables.
Constant of Variation (k)	The fixed number relating the variables; found from given data.

Key Concept: Direct Variation

METHOD 1. Write $y = kx$ (or $y = kx^n$ if stated as "varies directly as x^n ").
2. Substitute known values to solve for k .
3. Use the complete equation to answer the question.

EXAMPLE 1
 y varies directly as x . If $y=15$ when $x=3$, find k and the equation. Then find y when $x=8$.

$k = 15/3 = \rule{1cm}{0.4pt} \rightarrow y = \rule{1cm}{0.4pt} x \rightarrow \text{at } x=8: y = \rule{1cm}{0.4pt}$

Key Concept: Inverse Variation

EXAMPLE 2
 y varies inversely as x . If $y=4$ when $x=6$, find k and the equation. Then find y when $x=12$.

$k = xy = 6(4) = \rule{1cm}{0.4pt} \rightarrow y = \rule{1cm}{0.4pt} / x \rightarrow \text{at } x=12: y = \rule{1cm}{0.4pt}$

COMMON MISTAKE
For direct variation, $k = y/x$. For inverse variation, $k = xy$ (a PRODUCT, not a quotient). Mixing these up is the most common error in this chapter.

Key Concept: Joint & Combined Variation

EXAMPLE 3 — Joint Variation

z varies jointly as x and y . If $z=48$ when $x=4$ and $y=6$, find k . Then find z when $x=5$, $y=3$.

$$k = z/(xy) = 48/24 = \underline{\hspace{1cm}} \rightarrow z = \underline{\hspace{1cm}}xy \rightarrow \text{at } x=5, y=3: z = \underline{\hspace{1cm}}$$

EXAMPLE 4 — Combined Variation

y varies directly as x and inversely as z . If $y=10$ when $x=5$, $z=2$, find k . Then find y when $x=8$, $z=4$.

$$k = yz/x = 10(2)/5 = \underline{\hspace{1cm}} \rightarrow y = \underline{\hspace{1cm}}x/z \rightarrow \text{at } x=8, z=4: y = \underline{\hspace{1cm}}$$

Guided Practice — Part A: Direct Variation

- A1** y varies directly as x . $y=20$ when $x=4$. Find y when $x=9$.

- A2** y varies directly as x^2 . $y=45$ when $x=3$. Find y when $x=5$.

Guided Practice — Part B: Inverse Variation

- B1** y varies inversely as x . $y=8$ when $x=3$. Find y when $x=6$.

- B2** y varies inversely as x^2 . $y=5$ when $x=2$. Find y when $x=4$.

Guided Practice — Part C: Joint Variation

- C1** z varies jointly as x and y . $z=60$ when $x=5$, $y=4$. Find z when $x=3$, $y=7$.

- C2** V varies jointly as r^2 and h . $V=150$ when $r=5$, $h=6$. Find V when $r=3$, $h=10$.

Guided Practice — Part D: Apply It

D1 The force to stretch a spring varies directly with the distance stretched. A force of 20N stretches a spring 4cm. Find the force needed to stretch it 10cm.

D2 Light intensity varies inversely with the square of the distance from the source. Intensity is 100 lux at 2m. Find the intensity at 5m.

BEFORE YOU MOVE ON

Check with a partner: did you find k FIRST using the given data, before trying to answer the question?